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Heuristic Algorithm for Finding Sensitivity Analysis in Interval Solid Transportation Problems

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Abstract-This paper develops a heuristic algorithm for finding the ranges of cost in the interval solid transportation problem such that optimal basis is invariant. The procedure of the proposed approach is illustrated by numerical example.

Keywords- Interval solid transportation problem (ISTP), Cost sensitivity analysis

I. INTRODUCTION

The Solid transportation problem (STP) is an important augmentation of the transportation problem (TP). The STP arises when bounds are given on three items namely, supply, demand and conveyance. As a generalization of TP, the STP was introduced by Haley [7]. Pandian and Anuradha [14] have discussed new solution procedure for solving a STP. Sensitivity analysis (SA) is one of the most interesting and preoccupying areas in optimization. SA is to analyze the effect of the changes of the parameters in the optimization problems on the optimal value of the objective function as well as the validity ranges of these effects. SA for a linear programming problem were categorized and summarized by Koltai and Terlaky [11] and Hadigheh and Terlaky [5,6]. Doustdargholi et al. [3] discussed a new SA approach for RHS parameter in a TP. Ma and Wen [8] studied the cost coefficients SA of the degenerate TP. An algorithm for finding the SA of costs in a TP was presented by Lucia Cabulea [12]. Kavitha and Pandian [10] have introduced an algorithm for the cost SA in the STP. Jen et al. [2] discussed SA of the degenerate TP using labeling algorithm. ITP can arise when uncertainty exists in data problem and decision makers are more comfortable expressing it as intervals. Pandian and Anuradha [15] discussed the solution approach for ISTP. Badiya et al. [1] presented multi item interval valued STP with safety factor. Kavitha and Pandian [9] proposed an algorithm for solving SA of costs in ITP.

In this paper, a heuristic algorithm for finding the SA of costs in an ISTP is proposed and the same is illustrated with the help of numerical example. The SA of costs in an ISTP by the proposed algorithm can help the decision makers to determine what level of accuracy is necessary for a parameter to make the model sufficiently useful and valid when they are handling distribution problem having three constraints.

II. PRELIMINARIES

Let D denote the set of all closed bounded intervals on the real line R. That is, D= $\{ [a,b], a \le b \text{ and } a \text{ and } b \text{ are in } R\}$.

We need the following definitions of the basic arithmetic operators and partial ordering on closed bounded intervals which can be found in [4,13].

Definition 1: Let A = [a, b] and B = [c, d] be in D. Then,
(i) $A \oplus B = [a+c, b+d]$; (ii) $A \Theta B = [a-d, b-c]$;
(iii) $kA = [ka, kb]$ if k is a $+^{ve}$ real number; (iv) $kA = [kb, ka]$ if k is a $-^{ve}$ real number and
(v) $A \otimes B = [p, q]$ where p = min {ac, ad, bc, bd} and q = max {ac, ad, bc, bd}

Definition 2: Let A = [a, b] and B = [c, d] be in D. Then,
(i) $A \le B$ if $a \le c$ and $b \le d$; (ii) $A < B$ if $a < c$ and $b < d$;
(iii) $A \ge B$ if $B \le A$, that is $a \ge c$ and $b \ge d$ and (iv) $A = B$ if $A \le B$ and $B \le A$, that is, $a = c$ and $b = d$.

III. INTERVAL SOLID TRANSPORTATION PROBLEM

Consider the following ISTP:

$$\text{Minimize } [z_1, z_2] = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} [c_{ijk}, d_{ijk}] \otimes [x_{ijk}, y_{ijk}]$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{l}[x_{ijk}, y_{ijk}]=[a_i^1, a_i^2] \;,\; i=1,2,...,m \quad (1)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l}[x_{ijk}, y_{ijk}]=[b_j^1, b_j^2],\; j=1,2,...,n \quad (2)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}[x_{ijk}, y_{ijk}]=[e_k^1, e_k^2],\; k=1,2,...,l \quad (3)$$

$x_{ijk} \geq 0$, $y_{ijk} \geq 0$ for all *i, j* and *k* (4)

where c_{ijk}, d_{ijk}, a_i^1, a_i^2, b_j^1, b_j^2, e_k^1 and e_k^2 are positive real numbers for all i, j and k.

A set $\{[x_{ijk}, y_{ijk}]$, for all $i=1,2,...,m$, $j=1,2,...,n$ and $k=1,2,...,l$ $\}$ is said to be a feasible solution of the ISTP if they satisfy the equations (1), (2), (3)and (4).

A feasible solution of (ISTP) which minimizes the total shipping cost, that is, $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}[c_{ijk}, d_{ijk}]\otimes[x_{ijk}, y_{ijk}]$ is called an optimal solution (OS) to (ISTP).

We consider the following two problems as an upper bound (UB) problem and a lower bound (LB) problem of the given problem (ISTP):

(UB) Problem	(LB) Problem
Minimize $z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} d_{ijk}\, y_{ijk}$	Minimize $z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}\, x_{ijk}$
subject to $\sum_{j=1}^{n}\sum_{k=1}^{l} y_{ijk} = a_i^2, i=1,2,...,m$ …. (5) $\sum_{i=1}^{m}\sum_{k=1}^{l} y_{ijk} = b_j^2, j=1,2,...,n$ …. (6) $\sum_{i=1}^{m}\sum_{j=1}^{n} y_{ijk} = e_k^2, k=1,2,...,l$ …. (7) $y_{ijk} \geq 0$ for all i, j and k	subject to $\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = a_i^1, i=1,2,...,m$ …. (8) $\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = b_j^1, j=1,2,...,n$ …. (9) $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = e_k^1, k=1,2,...,l$ …. (10) $x_{ijk} \geq 0$ for all i, j and k

In [10] Pandian and Kavitha proved the following results which is used in the proposed algorithm.

Theorem 1 [10]: Let $(i, j, k)^{th}$ cell be a non-basic cell corresponding to an OS of the STP with $\delta_{ijk} = c_{ijk} - u_i - v_j - w_k (\geq 0)$. If $c_{ijk} + \Delta_{ijk}$ is the perturbed cost of c_{ijk}, then the range of $\Delta_{ijk} = [-\delta_{ijk}, \infty)$.

Theorem 2 [10]: Let $(i, j, k)^{th}$ cell be basic cell corresponding to an OS of the STP with $\delta_{ijk} = c_{ijk} - u_i - v_j - w_k (= 0)$. If $c_{ijk} + \Delta_{ijk}$ is the perturbed value of c_{ijk} and U_i is the minimum value of δ_{ijk} for all non-basic cells in the i^{th} origin, V_j is the minimum value of δ_{ijk} for all non-basic cells in the j^{th} destination and W_k is the minimum value of δ_{ijk} for all non-basic cells in the k^{th} conveyance, then the range of $\Delta_{ijk} = (-\infty, M_{ijk}]$ where M_{ijk} = the maximum $\{U_i, V_j, W_k\}$.

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IV. HEURISTIC ALGORITHM

A heuristic algorithm for finding the SA of ISTP is proposed below:

Step 1: Construct two individual problems of the given ISTP namely, (UB) problem and (LB) problem.
Step 2: Compute an OS to (UB) problem by the proposed method in [14].
Step 3: Create the MODI index matrix for the solution obtained in Step2:
(a) For all basic cells, use the relation $(u_i + v_j + w_k) = c_{ijk}$, and starting with any two MODI indices values zero, compute the remaining MODI indices.
(b) For all non-basic cells, compute $\delta_{ijk} = c_{ijk} - u_i - v_j - w_k$.
Step 4: Compute the cost ranges of all non-basic cells using the Theorem 1 [10] and then, compute the cost ranges of all basic cells using the Theorem 2 [10] to the (UB) problem.
Step 5: Repeat the steps from 2 to 4 for the (LB) problem with the upper bound constraints $x_{ijk} \le y^{\circ}_{ijk}$, for all i, j and k.

V. NUMERICAL EXAMPLE

The proposed method is illustrated by the following example.

Example 1: Consider the following ISTP:

										Capacity
Conveyance	E1			E1			E1			[26,33]
		E2			E2			E2		[13,18]
			E3			E3			E3	[14,17]
	D1			D2			D3			Supply
O1	f_{111}	f_{112}	f_{113}	f_{121}	f_{122}	f_{123}	f_{131}	f_{132}	f_{133}	[29,30]
O2	f_{211}	f_{212}	f_{213}	f_{221}	f_{222}	f_{223}	f_{231}	f_{232}	f_{233}	[8,12]
O3	f_{311}	f_{312}	f_{313}	f_{321}	f_{322}	f_{323}	f_{331}	f_{332}	f_{333}	[16,26]
Demand	[9,17]			[14,19]			[30,32]			[53,68]

where

f_{111}=[37,41]; f_{112}=[65,71]; f_{113}=[80,84]; f_{121}=[68,73]; f_{122}=[93,97]; f_{123}=[84,87]; f_{131}=[11,16];

f_{132}=[3,7]; f_{133}=[15,20]; f_{211}=[81,84]; f_{212}=[38,42]; f_{213}=[44,46]; f_{221}=[67,71]; f_{222}=[49,53];

f_{223}=[83,88]; f_{231}=[78,84]; f_{232}=[38,42]; f_{233}=[92,95]; f_{311}=[5,8]; f_{312}=[8,12];

f_{313}=[20,34]; f_{321}=[35,49]; f_{322}=[56,70]; f_{323}=[1,3]; f_{331}=[48,50]; f_{332}=[23,26]; f_{333}=[47,49].

Now, the (UB) problem of the given problem (ISTP) is given below:

										Capacity
Conveyance	E1			E1			E1			33
		E2			E2			E2		18
			E3			E3			E3	17
	D1			D2			D3			Supply
O1	41	71	84	73	97	87	16	7	20	30
O2	84	42	46	71	53	88	84	42	95	12
O3	8	12	34	49	70	3	50	26	49	26
Demand	17			19			32			

Now, using the proposed method in [14], the optimal solution to the (UB) problem is $y^{\circ}_{131} = 24$, $y^{\circ}_{132} = 6$, $y^{\circ}_{212} = 8$, $y^{\circ}_{222} = 2$, $y^{\circ}_{232} = 2$, $y^{\circ}_{311} = 9$ and $y^{\circ}_{323} = 17$.

Using Step 3, the MODI index matrix corresponding to the above solution is given below:

	E1			E1			E1			$w_1 = 51$
		E2			E2			E2		$w_2 = 42$
			E3			E3			E3	$w_3 = 35$
	D1			D2			D3			
O1	25	64	84	46	79	76	0	0	20	$u_1 = -35$
O2	33	0	11	9	0	42	33	0	60	$u_2 = 0$
O3	0	13	42	30	60	0	42	27	57	$u_3 = -43$
	$v_1 = 0$			$v_2 = 11$			$v_3 = 0$			

Using Step 4, the range of the (UB) problem are given below:

	E1			E1			E1		
		E2			E2			E2	
			E3			E3			E3
	D1			D2			D3		
O1	$[-25,\infty)$	$[-64,\infty)$	$[-84,\infty)$	$[-46,\infty)$	$[-79,\infty)$	$[-76,\infty)$	$(-\infty,20]$	$(-\infty,20]$	$[-20,\infty)$
O2	$[-33,\infty)$	$(-\infty,13]$	$[-11,\infty)$	$[-9,\infty)$	$(-\infty,13]$	$[-42,\infty)$	$[-33,\infty)$	$(-\infty,20]$	$[-60,\infty)$
O3	$(-\infty,13]$	$[-13,\infty)$	$[-42,\infty)$	$[-30,\infty)$	$[-60,\infty)$	$(-\infty,20]$	$[-42,\infty)$	$[-27,\infty)$	$[-57,\infty)$

Now, the (LB) problem of the given problem (ISTP) with the upper bound constraints is given below.

										Capacity
Conveyance	E1			E1			E1			26
		E2			E2			E2		13
			E3			E3			E3	14
	D1			D2			D3			Supply
O1	37	65	80	68	93	84	11	3	15	29
O2	81	38	44	67	49	83	78	38	92	8
O3	5	8	20	35	56	1	48	23	47	16
Demand	9			14			30			

and $x_{ijk} \le y^{\circ}_{ijk}$, $i = 1,2,...,m$; $j = 1,2,...,n$ and $k = 1,2,...,l$.

Using the procedure followed as in the solution of (UB) problem, the range of the (LB) problem is obtained as

	E1			E1			E1		
		E2			E2			E2	
			E3			E3			E3
	D1			D2			D3		
O1	$[-26,\infty)$	$[-62,\infty)$	$[-84,\infty)$	$[-46,\infty)$	$[-79,\infty)$	$[-77,\infty)$	$(-\infty,19]$	$(-\infty,19]$	$[-19,\infty)$
O2	$[-35,\infty)$	$(-\infty,11]$	$[-13,\infty)$	$[-10,\infty)$	$(-\infty,11]$	$[-41,\infty)$	$[-32,\infty)$	$(-\infty,19]$	$[-61,\infty)$
O3	$(-\infty,11]$	$[-11,\infty)$	$[-30,\infty)$	$[-19,\infty)$	$[-48,\infty)$	$(-\infty,13]$	$[-43,\infty)$	$[-26,\infty)$	$[-57,\infty)$

VI. CONCLUSION

In this paper, we obtained the perturbation range of costs SA of ISTP. The necessity of considering cost SA of the ISTP arises when heterogeneous conveyances are available for shipment of products in public distribution systems. The proposed method can help the decision makers to determine what level of accuracy is necessary for a parameter to make the model sufficiently useful and valid when they are handling distribution problem having three constraints.

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