Intensity-Duration-Frequency Curves and Regionalisation

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Abstract* – Storm sewers make up a large percentage of drainage system in an urban setup. The design of these components are based on rainfall intensities of a specific design period for that location. These can be derived from intensity-duration-frequency (IDF) relationship. These IDF relationships are derived from historical rainfall, using an extreme value distribution for maximum rainfall intensity. In the present study the IDF curves and parameter regionalisation were studied for various kinds of basins. These equation parameters can be then used to understand the spatial variation of rainfall intensity in the study area. The parameter contour maps subsequently generated using various interpolation method are then used for plotting IDF curves for any ungauged station in the basin.

Keywords: IDF Curves, Urban Rainfall, Variation, Regionalisation

I. INTRODUCTION

The rainfall Intensity-Duration-Frequency (IDF) relationship is one of the most commonly used tools in water resources engineering, either for planning, designing and operating of water resource projects, or for various engineering projects against floods [1]. The establishment of such relationships were done as early as in 1932 (Bernard). Since then, many sets of relationships have been constructed or several parts of the globe.

The following methods are generally used for estimating runoff during a storm: 1). Rational method 2). Empirical method 3). Unit-hydrograph technique 4). Flood-frequency studies. The use of a particular method depends upon (i).the desired objective, (ii). The available data, and (iii) the importance of the project. Further, the rational formula is only applicable to small-size (< 50 km^2) catchments and the unit-hydrograph method is normally restricted to moderate-size catchments with areas less than 5000 km^2.The empirical formulae that are available for estimating the storm water runoff can be used only when comparable conditions to those for which the equations were derived initially can be assured.

A rational approach, therefore, demands a study of the existing precipitation data of the area concerned to permit a suitable forecast. Storm sewers are not designed for the peak flow of rare occurrence such as once in 10 years or more but, it is necessary to provide sufficient capacity to avoid too frequent flooding of the drainage area. There may be some flooding when the precipitation exceeds the design value, which has to be permitted. The frequency of such permissible flooding may vary from place to place, depending on the importance of the area. Though such flooding causes inconvenience, it may have to be accepted once in a while considering the economy effected in storm drainage costs.

The maximum runoff which has to be carried in a sewer section should be computed for a condition when the entire basin draining at that point becomes contributory to the flow and the time needed for this is known as the time of concentration(t_c) with reference to the concerned section. Thus, for estimating the flow to be carried in the storm sewer, the intensity of rainfall which lasts for the period of time of concentration is the one to be considered contributing to the flow of storm water in the sewer. Of the different methods, the rational method is more commonly used. The runoff reaching the sewer is given by the expression, [2]

$$Q = ciA$$

Where,

Q=peak flow required for sizing storm sewers;
c =runoff coefficient which is usually empirically determined based on land use and soils of the catchment;
i=intensity of rainfall of chosen return period for a duration equalling the time of concentration of the catchment;
A=area of the catchment.

The Intensity of Rainfall to be used in computing the design run-off in the equation is related to the concentration time and the frequency of occurrence of the storm. The frequency of occurrence has to be selected by the designers at the start of the design exercise. The costof the system depends upon this decision. Considering a rarer storm will increase the cost of the system because the intensity of rainfall will be high. Table 1 shows the C.P.H.E.E.O Manual design frequencies:

A point to point design of the Storm Water Drainage System requires repeated selection of the Intensity of Rainfall for increasing Concentration Time intervals. This is possible if the relationship of the Intensity of Rainfall with the concentration time for a given Frequency of Occurrence is known for the particular catchment. This relationship is known as Intensity –Duration – Frequency (IDF) relationship and can be expressed in the form of a formula or in the form of a graphic curve. Such a relationship can be developed by conducting an analysis of past rainfall records for this area under study.

TABLE I RETURN PERIODS RECOMMENDED BY C.P.H.E.E.O [3].

Type of Area	Frequency
Residential Areas	
1 Peripheral Areas	2 in 1
2 Central and comparatively high prices areas	1 in 1
Commercial and High Priced Areas	1 in 2

The use of rainfall Intensity-Duration-Frequency relationships (IDF relationships) has been standard practice for many decades for the design of sewer systems and other hydraulic structures. The IDF relationships give an idea about the frequency or return period of a mean rainfall intensity or rainfall volume that can be expected within a certain period, i.e. the storm duration. In this sense the storm duration is an artificial parameter that can comprise any part of a rainfall event. Even in this computer age they provide a lot of information on the rainfall and they can be used as a base for the determination of design storms. The reason is the physically based link between IDF relationships and hydraulic design, i.e. the concentration time. Fig. 1 shows a typical IDF curve for Egypt

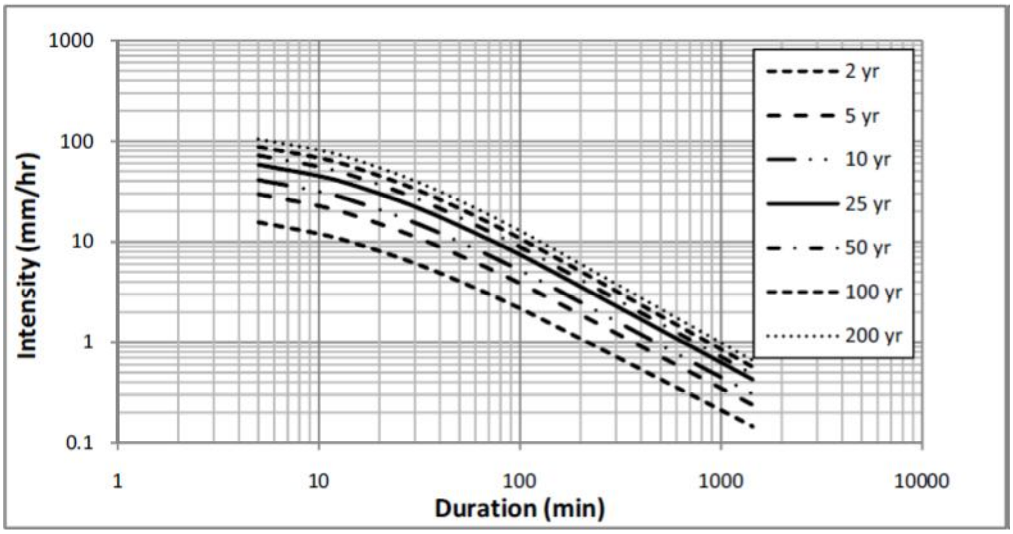

Fig. 1 A Typical IDF Curve (Source: El-Sayed, 2011)

The various components of an urban drainage system are designed to have enough capacity for the conveyance of storm water resulting from heavy rainfalls of specified frequencies of occurrence. These specified frequencies of occurrence (often expressed in return periods) thus constitute the hydrologic and hydraulic design standards (or criteria) of urban drainage systems. Rainfalls heavier than those of the design criteria may result in sewer backup causing basement flooding, localized street flooding, as well as larger area flooding. With the observed and predicted increase in the frequency of heavy rainfall events, the capacity of urban drainage systems designed according to outdated heavy rainfall frequency information would be inadequate during their long service lives. The degree of this inadequacy needs to be better understood so that corrective or adaptive actions may be taken in a timely fashion. The purpose of this study is to raise awareness and demonstrate the need for up to date heavy rainfall frequency information.

Reference [1] enlists four equations which can depict an intensity-duration-frequency curves viz, Talbot, Bernard, Kimijima, Sherman and the method given by Fair & Geyer which was used in analysis for Mumbai [4]. The Fair & Geyer method formula is basically modified Sherman’s formula. Normally Bernard’s & Sherman’s equation are developed for a specific frequency of occurrence and the parameter constants are applicable to respective frequency. In the relationship given in Fair & Geyer, the values of the set of constants C, m, b and e are the same for frequencies lying in a given range, for other ranges the values will vary.

II. METHODS OF SOLUTION FOR IDF EQUATIONS

Three methods are generally used for solving the IDF equations, a Graphical Method, method of Annual Maxima Series, as well as a method based on Least Squares Principle are available to find out the magnitudes of the constants

For mathematical modelling purposes, it is convenient to express average intensity-duration-return period relationships in the form of a simple equation as

$$i = \frac{a(T)}{b(d)}$$

For a specified return period the above equation can be modified to

$$i = \frac{a(T)}{(d + x)^y}$$

Where a, x and y are parameters to be estimated. These are usually found out by using an appropriate frequency distribution functions. In case of intensity-duration-frequency analysis, the following are generally used [5]

1. Gumbel's extreme-value distribution,
2. Generalised Extreme Value (GEV) distribution
3. Gamma distribution
4. Log-Pearson Type III distribution
5. Log Normal distribution.
6. Exponential distribution
7. Pareto's distribution

The type I distribution of maxima, also termed the Gumbel distribution function, is the most widely used distribution for IDF analysis due to its suitability for modelling maxima. [5] Given that the rainfall intensity I(d) has Gumbel distribution for any duration d, so will have Y and which is an exact yet simple expression of a(T).

$$y_T \equiv a(T) = \lambda \left[\psi - ln\left\{-ln\left(1 - \frac{1}{T}\right)\right\}\right]$$

In all distributions, κ and ψ denote dimensionless parameters whereas λ and c denote parameters having same dimensions as the random variable y (or ln y in case of logarithmic transformation of the variable).

Reference [1] proposed an approach for the formulation and construction of IDF curves using data from recording station by using empirical formulae/equation and the comparison between the given formulae and choosing the best equation that could be representative for Vietnam. The method proposed in the study was reasonably applicable for ungauged rainfall locations, which was verified by checking with additional raingauge station. Analysis of distribution for rainfall frequency was based on the Pearson Type III distribution, which iscommonly used in Vietnam for this kind of analysis

Reference [6], developed IDF curves and isopluvial maps for six different durations of 5, 15, 30, 60, 120 and 1440 minutes) and seven different return periods of 2, 5, 10, 25, 50, 100 and 200 years in the Arab Republic of Egypt. Also, the paper proposes the approach for formulation and construction of IDF curves using data fromrecording station by using empirical equation and determining the equation parameters for the region.

Reference [7] studied rainfall data of 14 raingauge stations of Punpun basin located in Bihar and regional rainfall frequency analysis based on L-moment approach facilitated to find the robust distribution for these daily raingauge stations. Therobust distribution was used to find the IDF relationship and curves for short duration rainfall for Punpun basin. From the IDF curves parameters of empirical equations for the gaugedlocations were determined. The study revealed that out of the following distributions, viz. Generalized Extreme Value, Generalized Logistic, Generalized Normal,Pearson Type-III; it was found that Generalized Logistic distribution was identified as the robust distribution for Punpun basin

III. REGIONALISATION

After determining the parameters of IDF formula such as parameters a, b and parameter e, (C, m ,b and e, in case of Fair & Geyer Method), for the same return period, parameter contour maps can generated for the parameters which can then be used for ungauged rainfall with return periods, using various geo-statistical interpolation tools. The regional IDF formula parameters are generated for ungauged areas to estimate rainfall intensity for various return period and rainfall duration. The method reasonable application to ungauged rainfall location, which is concluded from the verification of additional raingauges.

The various methods used for parameter regionalisation are as follows:

- Trend surface analysis,
- Triangulation with smoothening
- Thin-plate splines,
- Inverse distance weighting average,
- Ordinary kriging algorithm
- Universal kriging algorithms

The trend surface analysis algorithm simply uses multiple regression (i.e., least squares) to fit a polynomial surface to the entire set of observations. Since final interpolated surfaces represent the broad regional trend in the variation implied by the data, and seldom pass exactly through the original data points, the algorithm is typically referred to as an inexact interpolator. These techniques usually assume that rainfall IDF estimates are normally distributed, which may not be the case.

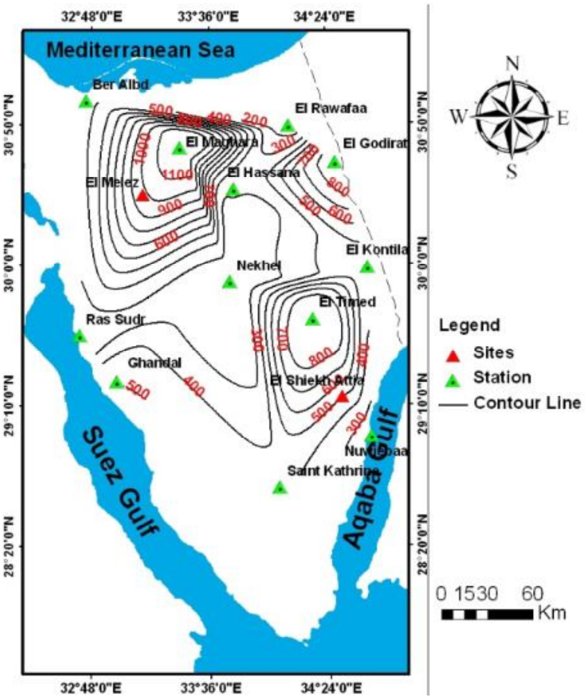

FIG. 2 TRIANGULATION WITH SMOOTHENING, PARAMETER A (SOURCE: EL-SAYED, 2011)

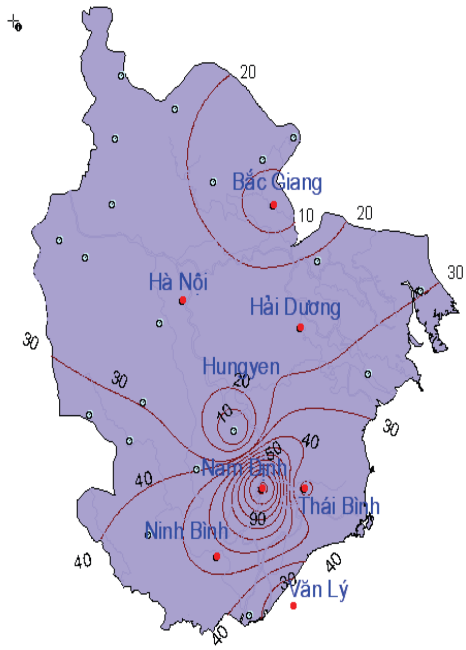

FIG. 3 THIN-LINE SPLINE INTERPOLATION FOR PARAMETER B IN VIETNAM (SOURCE: NHAT 2006)

The appropriateness of several spatial interpolation algorithms for developing isopluvial maps can be based on point or regional IDF estimates. For practical reasons, the assessment was limited to commonly used algorithms such as those available in a commercial spatial interpolation software (Matlab, R) package or GIS.

Thin-plate splines also use multiple regression to fit a polynomial surface to a set of observations. However, the algorithm performs piece-wise multiple regression on subsets of observations that are within a predefined neighbourhood, which allows local variation in the observations to be retained in the final interpolated surface. Splines follow the general notion that interpolated values are similar to the nearest known observation values.

The advantage of using thin-plate splines with respect to rainfall IDF estimates is that only observations within a predefined vicinity influence interpolated estimates. The disadvantages of thin-plate splines are that there is no method for selecting an appropriate neighbourhood radius size, and there is no direct method for assessing the error in interpolated values. Also, interpolated surfaces may contain unrealistic highs and lows as a result of exact interpolation.

Inverse distance weighting averages follow the principle that nearby observation points are more influential than distant observations. The inverse distance weighting algorithm involves the following steps: 1) select the nearest observation points within a predefined neighbourhood; 2) assign a weight to each observation based on its distance from the interpolation point; 3) sum the weighted observations to find an average. Due to the mathematical nature of inverse distance weighting, the algorithm is an exact interpolator.

The advantages of using inverse distance weighting for isopluvial maps are that the interpolated surface is confined within the range of the point estimate values, and only observations within a predefined vicinity influence the interpolated estimates. Disadvantages of the algorithm are that there are no methods of testing the quality of predictions, the final interpolated surface is strongly dependent on the size of the neighbourhood radius and dominated by local variation, and the surface slope may be discontinuous at observation points as a result of exact interpolation.

Kriging is the geostatistical method of interpolation that provides the best, i.e., minimum variance, unbiased linear estimates of a set of observations. Similar to inverse distance weighting, the kriging algorithm models spatial variation by assigning weights to the observations within a predefined search neighbourhood. However, unlike the previously mentioned algorithms, kriging also provides a theoretical basis for determining the most appropriate weights and size of the search neighbourhood.

Kriging optimizes spatial interpolation based on the assumption that spatial variation consist of three major components: 1) a structural component, having a constant mean or trend; 2) a random, but spatially correlated component; and 3) a spatially uncorrelated component (random noise or residual error). This treatment of the variation is known as the regionalized variable theory, which is based on the intrinsic hypothesis that assumes once structural effects have been accounted for, the remaining variation is homogeneous, i.e., differences between sites are only a function of distance. Although there are several variations of kriging, only ordinary and universal kriging will be assessed in this paper for developing isopluvial maps. Ordinary kriging provides a means of interactively investigating the spatial variation.

Advantages of using ordinary kriging for mapping rainfall IDF estimates are that the use of a semivariogram model assists in the selection of appropriate interpolation control parameters, and the algorithm provides a direct estimate of the quality of the interpolated surface in terms of the estimation variance. Disadvantages of ordinary kriging are that the selection of the appropriate semivariogram model, the lag distance, and the grid size for the interpolated surface introduces some degree of subjectivity and requires knowledge of geostatistical theory. [8]

GIS softwares such as ArcGIS are powerful tools, but are expensive license based softwares. There are free statistical softwares like R Programming where various packages like *gstat* and *geoR* are available. These are powerful tools and give all kinds of interpolation techniques mentioned above. Disadvantage of programming statistical program requires an in-depth knowledge of geo-statistical computing R is an environment within which many classical and modern statistical techniques can be implemented. R is a vehicle for newly developing methods of interactive data analysis. It has has an extended and large collection of packages. There are about 25 packages supplied with R (called “standard” and “recommended” packages) and many more are available through the CRAN family of Internet sites (via http://CRAN.R-project.org) and elsewhere. [9]

IV. CONCLUSIONS

A set of IDF curves are generated for a given rainfall station and the unknown parameters are unique to that individual station. It is seen from various studies that there indeed is spatial variation in the rainfall intensity in an urban area, it would be more sensible to plot IDF curve for various raingauge station located in an urban agglomeration. If these relationships can be derived at an ungauged point, the design value could be nearer to the actual values. Also, the study area is still undergoing development; various infrastructure works (storm water drainage network) could make use of regionalised IDF curves.

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